ANALOGY OF CONVECTIVE TRANSFER PROCESSES WITH DIFFERENT BULK SOURCES IN THE BOUNDARY LAYER

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UDC 526.532

The Reynolds analogy, based on the similar nature of the heat transfer and momentum processes, permits finding the thermal flux from a surface on the basis of the solution of the motion equation. It is well known in boundary-layer theory. The presence of longitudinal pressure gradient in the flux results in spoilage of the Reynolds analogy. The pressure gradient in the boundary layer can be considered as an additional momentum "source." The appearance of sources of different physical nature that have a similar formal description in a flow can result in similarity of the transport equations, but this, in turn, will result in similarity of their solutions.

In connection with the difficulty in studying transfer processes in reacting fluxes, the attention of researchers has turned to the qualitative analogy between the momentum and heat transfer processes in flows with sources [1, 2]. Using the enhanced experience of a theoretical and experimental study of gradient flows, the development of methods to analyze chemically nonequilibrium flows can be simplified substantially.

By theoretically giving a foundation to the existence of the quantitative analogy and by determining its boundaries, the validity of mathematical models can be confirmed not only for laminar, but also for turbulent flows with chemical reactions or other heat liberation sources in a flow; also the establishment of certain quantitative regularities for such flows can be attempted.

Let us examine two systems of turbulent boundary-layer equations describing the processes of momentum and heat transfer with volume sources in dimensionless form by relying

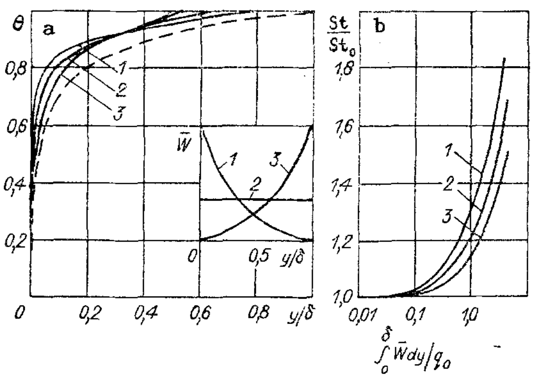

Fig. 1

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 139-143, January-February, 1988. Original article submitted October 3, 1986.

on the hypothesis of turbulent viscosity. The systems include the equations of the balance of the fluctuation intensity of the longitudinal velocity and temperature components written in conformity with [3].

The equations of a turbulent boundary layer with a pressure gradient are

$$\frac{\partial \widetilde{\rho u}}{\partial \tilde{x}} + \frac{\partial \widetilde{\rho v}}{\partial \tilde{y}} = 0; \tag{1}$$

$$\tilde{\rho}\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{\rho}\tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = \frac{1}{\mathrm{Re}}\frac{\partial}{\partial \tilde{y}}\left[\tilde{\rho}\left(\tilde{\nu}+\tilde{\nu}_T\right)\frac{\partial \tilde{u}}{\partial \tilde{y}}\right] + \widetilde{W}_1; \tag{2}$$

$$\tilde{\rho}\tilde{u}\frac{\partial \tilde{u}'^2}{\partial \tilde{x}} + \tilde{\rho}\tilde{v}\frac{\partial \tilde{u}'^2}{\partial \tilde{y}} = \frac{1}{\mathrm{Re}}\frac{\partial}{\partial \tilde{y}}\left[\tilde{\rho}\left(\tilde{\nu}+\frac{\tilde{\nu}_T}{\gamma_1}\right)\frac{\partial \tilde{u}'^2}{\partial \tilde{y}}\right] + \frac{2}{\mathrm{Re}}\tilde{\rho}\tilde{\nu}_T\left(\frac{\partial \tilde{u}}{\partial \tilde{y}}\right)^2 - \frac{2}{\mathrm{Re}}A_1\left(\tilde{\nu}+\alpha_1\tilde{\nu}_T\right)\frac{\tilde{u}'^2}{\tilde{l}^2} + \tilde{Q}_1. \tag{3}$$

The equations describing turbulent heat transfer with a volume heat source are

$$\frac{\partial \widetilde{\rho u}}{\partial \tilde{x}} + \frac{\partial \widetilde{\rho v}}{\partial \tilde{y}} = 0,\ \tilde{\rho}\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{\rho}\tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = \frac{1}{\mathrm{Re}}\frac{\partial}{\partial \tilde{y}}\left[\tilde{\rho}\left(\tilde{\nu}+\tilde{\nu}_T\right)\frac{\partial \tilde{u}}{\partial \tilde{y}}\right]; \tag{4}$$

$$\tilde{\rho}\tilde{u}\frac{\partial \tilde{\Theta}}{\partial \tilde{x}} + \tilde{\rho}\tilde{v}\frac{\partial \tilde{\Theta}}{\partial \tilde{y}} = \frac{1}{\mathrm{Pe}}\frac{\partial}{\partial \tilde{y}}\left[\tilde{\rho}\left(\tilde{\nu}+\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\tilde{\nu}_T\right)\frac{\partial \tilde{\Theta}}{\partial \tilde{y}}\right] + \widetilde{W}_2; \tag{5}$$

$$\begin{gathered}\tilde{\rho}\tilde{u}\frac{\partial \tilde{\Theta}'^2}{\partial \tilde{x}} + \tilde{\rho}\tilde{v}\frac{\partial \tilde{\Theta}'^2}{\partial \tilde{y}} = \frac{1}{\mathrm{Pe}}\frac{\partial}{\partial \tilde{y}}\left[\tilde{\rho}\left(\tilde{\nu}+\frac{\mathrm{Pr}}{\mathrm{Pr}_T\gamma_2}\tilde{\nu}_T\right)\frac{\partial \tilde{\Theta}'^2}{\partial \tilde{y}}\right] + \\ + \frac{2}{\mathrm{Pe}}\left[\tilde{\rho}\tilde{\nu}_T\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\left(\frac{\partial \tilde{\Theta}}{\partial \tilde{y}}\right)\right]^2 - \frac{2}{\mathrm{Pe}}A_2\left(\tilde{\nu}+\alpha_2\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\tilde{\nu}_T\right)\frac{\tilde{\Theta}'^2}{\tilde{l}^2} + \tilde{Q}_2.\end{gathered} \tag{6}$$

The equations are written for quantities averaged with respect to time. Here $\tilde{\rho} = \frac{\rho}{\rho_\infty}$; $\tilde{u} = \frac{u}{u_\infty}$; $\tilde{v} = \frac{v}{v_\infty}$; $\tilde{\nu} = \frac{\nu}{\nu_\infty}$; $\tilde{\nu}_T = \frac{\nu_T}{\nu_\infty}$; $\tilde{\Theta} = \frac{T - T_w}{T_\infty - T_w}$; $\mathrm{Re} = \frac{u_\infty L}{\nu_\infty}$; $\mathrm{Pe} = \mathrm{Re}\,\mathrm{Pr}$; $\tilde{x} = \frac{x}{L}$; $\tilde{y} = \frac{y}{L}$; L is the characteristic dimension of the body, $\tilde{\ell} = \ell/L$ is the dimensionless turbulence scale; $\widetilde{W}_1 = -\frac{L}{\rho u_\infty^2}\frac{dp}{dx}$, $\widetilde{W}_2 = \frac{L}{\rho u_\infty (T_\infty - T_w)} W_2$ are the dimensionless momentum and heat sources; $\tilde{u}'^2 = \langle u'^2\rangle/u_\infty$; $\tilde{\Theta}'^2 = \langle T'^2\rangle/(T_\infty - T_w)^2$; $\tilde{Q}_1 = \langle \tilde{W}_1 u'^2\rangle$, $\tilde{Q}_2 = \langle \tilde{W}_2 \tilde{\Theta}'^2\rangle$ are additional correlations that originate in the fluctuation intensity balance equations upon the appearance of source terms in the average equations; and A_{1-2}, α_{1-2}, γ_{1-2} are constants [3].

The systems of equations (1)-(3) and (1), (4)-(6) have identical boundary conditions $\tilde{y} = 0$, $\tilde{u} = 0$, $\tilde{\Theta} = 0$, $\tilde{u}'^2 = 0$, $\tilde{\Theta}'^2 = 0$; $\tilde{y} \to \infty$, $\tilde{u} \to 1$, $\tilde{\Theta} \to 1$, $\tilde{u}'_2 \to 0$, $\tilde{\Theta}'^2 \to 0$ for a given wall temperature and small degree of turbulence in the external flow.

The differences in the equations (2), (3) from (4), (6) can be the following for identical values of the governing parameters Re, Pe: different values of the velocities $\tilde{u}$ and $\tilde{v}$ enter into the convective terms; the source terms $\tilde{W}_1$ and $\tilde{W}_2$ can have different transverse and longitudinal distributions; the additional terms $\tilde{Q}_1$ and $\tilde{Q}_2$ appearing in the fluctuation equations are different (even under conditions of an identical distribution in the $\tilde{W}_1$ and $\tilde{W}_2$ flux); the turbulent heat and momentum transfer coefficients are different. Nevertheless, it can be shown that under definite conditions, the differences listed will not result in significant discrepancies in the solutions.

It is shown experimentally in [4] that the pressure gradient, which strongly fluences the velocity distribution in the flow, the friction coefficient, and the boundary-layer thickness, changes the heat transfer and the temperature distribution slightly. Computations performed in this paper confirm the regularity noted. Starting from this, the deduction can be made that a limited pressure gradient $\left(\frac{x}{u_\infty}\frac{du_\infty}{dx}<0.1\right)$ influences the convective heat transfer slightly in a turbulent boundary layer, and the difference between the convective components in (2) and (5) has slight effect on their solutions. The distinctive nature of the change in the convective terms has slight influence on the solution of (3) and (6) because of the limited role of the convection in the balance of the turbulence kinetic energy intensity [3] and the temperature fluctuation.

In order to clarify the influence of the source transverse distribution mode on the common nature of the flow and the heat transfer, the solutions of (1), (4)-(6) are compared for sources of different shape but identical integrated value: with greatest heat liberation at the wall, with uniform distribution in the boundary layer, with greatest heat liberation at the outer boundary-layer boundary (curves 1-3 in Fig. 1a). The turbulent viscosity was calculated on the basis of the Prandtl hypothesis. The mixing path was computed by using the Simpson and Van Driest corrections [5]. The turbulent Prandtl number was taken equal to unity. The solution is obtained by the local self-similarity method [8].

Represented in Fig. 1 are the results of computations of the temperature profile and the Stanton number, St. Curves corresponding to identical sources are marked with numbers. The computed temperature distribution in the boundary layer (without volume heat liberation) is shown by dashes in Fig. 1a. It is seen that a significant change in the source shape influences the boundary-layer characteristics slightly. The sign and integrated value of the source, and more accurately, the ratio between this integral and the heat flux at the wall, play the governing role.

The integral of the source with respect to the boundary-layer thickness equals that additional momentum, heat, or mass of the chemical component manifested in the boundary-layer section under consideration. This latter is transferred downstream either by convective means or to the wall because of molecular and turbulent transfer. The quantity of heat or momentum additionally incident on the wall determines the corresponding velocity or temperature gradient. The mode of the source distribution also influences the solution, since the characteristic distance between the wall and the maximal heat liberation zone depends on it. However, as computations show, this influence becomes perceptible only for sufficiently large differences in the source distribution mode (the maximal values differ by more than two times). As the results of solving the system (1) and (6) and a number of other investigations [5, 6] show, the nature of the longitudinal change in typical chemical sources in similarity variables exerts small influence on the solution of the problem. It is seen from the computations that if the heat flux (momentum, mass of the component) is directed to the wall, then the presence of a source of positive sign in the boundary layer will result, independently of its mode, in an increase in the heat flux (momentum, mass of the chemical component) at the wall, in a diminution in the thermal (hydrodynamic, concentrated) boundary-layer thickness, in a growth of the population of the profiles of the corresponding average characteristics, and in a diminution of the fluctuation characteristics in the boundary layer.

The appearance of a negative source or a change in the heat flux, momentum, component mass in the stream will result in reverse results. The mentioned regularities are confirmed for the presence of a pressure gradient in the boundary layer by all known experimental data in both the average and the fluctuation characteristics ([7], et al).

Figures 2 and 3 show a comparison of the results of a computation by the method of [8] for the average and fluctuation turbulent boundary-layer characteristics, respectively: a hydrodynamic (with positive pressure gradient) described by the system (1)-(3), and a thermal described by the system (1), (4)-(6) with a chemical source of the form $\tilde{W}_2$ = Da (exp (10(y − 1)) − Θ^2). Data on the nonequilibrium thermal boundary layer are presented as a function of the Damkeller number, Da, which characterizes the degree of nonequilibrium flow and determines the thermal effect of the reaction. Experimental results on the gradient flows are shown by dashed lines [7].

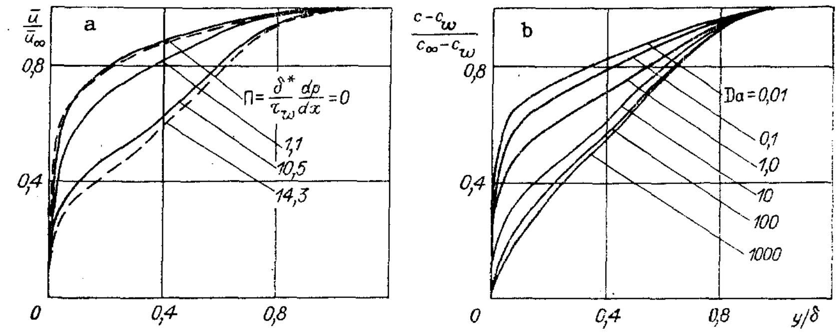

Fig. 2

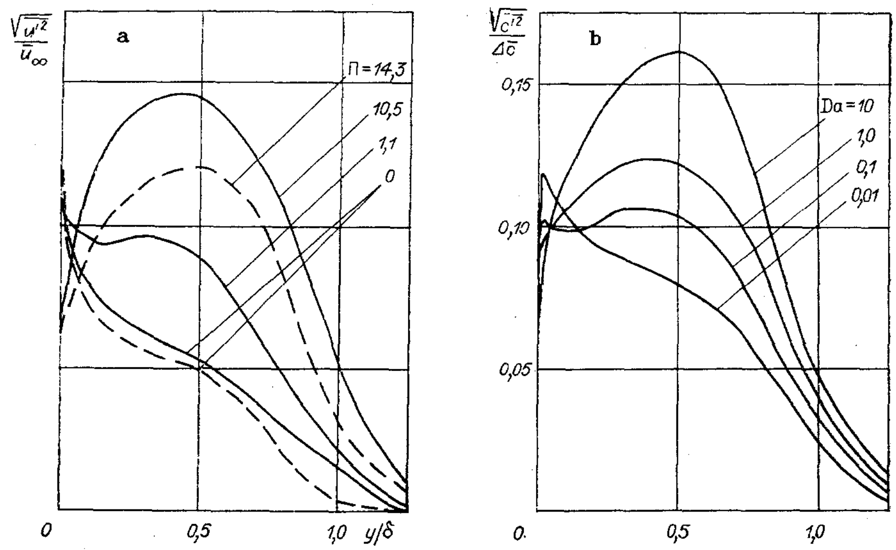

Fig. 3

The materials represented in Figs. 2 and 3 indicate the existence of a broad range of a "similar" change in the characteristics of hydrodynamic and thermal boundary layers with sources of different nature.

Let us examine the additional correlations that appear in (3) and (6). Since $W_1(\tilde{u})$ and $W_2(\tilde{\Theta})$ can have a distinct nature, Q_1 and $\tilde{Q}_2$ should be distinct even for an identical $\tilde{u}$ and $\tilde{\Theta}$ distribution in the stream. However, many cases of practical importance exist when these correlations are negligibly small. Among chemically reacting flows, a broad class of flows, nonequilibrium at the average level ($\tau_{dyn} \sim \tau_{chem}$) and frozen at the fluctuation level, satisfy this condition since the inequality $\tau_{tur} \ll \tau_{dyn}$ is valid.

Analysis of the second moment equations for the Reynolds stresses and turbulent heat fluxes results in the deduction about the similarity of the turbulent viscosity and the thermal conductivity coefficients because of the similarity of the fields of average values and velocity and temperature fluctuations of the two problems under consideration. This latter does not mean equality of the turbulent Prandtl number to unity in the thermal problem since different values of the turbulent viscosity coefficients enter into the corresponding systems (1)-(3) and (4)-(6).

Let us list the conditions under which the analogy under consideration exists:

Equality of the governing parameters Re and Pe;

Identical boundary conditions;

Identical sign and equality of the integrated values of the sources over the boundary-layer thickness;

No significant differences in the source distribution mode (maximal values differ by not more than two times);

Relative smallness of the additional correlations in the second moment equations.

Upon satisfaction of these conditions, the analogy under consideration affords a possibility of analyzing convective thermal and mass transfer with volume sources on the basis of tests on gradient flows.

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HEAT TRANSFER DURING MIXED CONVECTION ON A VERTICAL SURFACE IN A POROUS MEDIUM WITH DEVIATION FROM DARCY'S LAW

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UDC 536.255-66.067.1

In recent years, the requirements of modern technology have stimulated interest in the study of flows which involve the interaction of several phenomena. One such problem is heat transfer during mixed natural and forced convection in porous media. The need to solve these problems stems from the broad use of granular media in chemical engineering (granular beds of catalysts) and the use of geothermal power energy sources and methods of intensifying oil and gas extraction, which are based either on organizing a moving combustion source or pumping hot water or steam. The problems are also encountered in the use of heat pipes and other devices.

In these cases, convective heat flows are realized in porous media when a heated (or cooled) object is placed in a fluid whose density changes with temperature. Forced convection occurs when an external flow moves around a surface.

Problems of heat transfer with free and forced convection in a Darcy's law approximation have been studied in the greatest detail to date. Investigators have examined heating surfaces with different geometries (plate, cylinder, flow along the inside surface of a cylinder) and different orientations in space — vertical, horizontal, and inclined plates. A detailed survey of the problems studied is offered in [1]. The problems were solved in a boundary-layer approximation and are based on the Darcy flow model. Conditions were established for the existence of similarity solutions for corresponding methods of assigning boundary conditions, and relations were found for the exponents in power laws describing the distributions of the external flow and wall temperature.

However, we should point out the rather narrow range of applicability of Darcy's law [2]. It is restricted to the Reynolds number limit $Re = u\sqrt{\Pi}/\nu \le O(1)$, constructed from the

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 143-149, January-February, 1988. Original article submitted July 28, 1986.

0021-8944/88/2901-0133$12.50

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